

Image denoising based on wavelet analysis and quantum-behaved particle swarm optimization

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Abstract

This paper investigates the basic principle of threshold denoising based on wavelet transform, including the selection of wavelet basis, the determination of wavelet decomposition level, the selection method of threshold and the threshold estimation method of wavelet coefficient. Additionally, it proposes an image denoising method based on quantum-behaved particle swarm optimization (QPSO), gives the optimization value based on the experiments and theoretical analysis and optimizes the dynamic threshold by using numerous advantages of wavelet transform in the field of image denoising and QPSO so as to realize the self-adaptive denoising of wavelet transform and reduce the influence of subjective factors. The simulation experiment shows that in addition to the effective denoising, the algorithm of this paper protects the image details and obtains better image denoising effects.

Keywords: wavelet analysis, quantum-behaved particle swarm optimization (QPSO), image denoising

1 Introduction

The image acquisition and transmission are often disturbed by various noises. The image denoising effects usually directly affect the subsequent image processing. When using the traditional denoising methods, side effects such as image blur appear and it is not ideal enough to maintain the image details. In recent years, many techniques including wavelet analysis idea and intelligent optimization algorithms have gradually been applied in digital image processing and they have achieved better denoising effects. Due to such advantages as time-frequency analysis, multi-resolution and ease to match with human vision characteristics, wavelet analysis is widely used in the fields like edge detection, image compression and image denoising [1].

Intelligent optimization algorithms are the algorithms that human beings construct the optimization process by simulating the biological system in the natural world. Based on group iteration, in the particle swarm optimization (PSO), the group searches following the optimal particle in the group space in order to integrate global optimization and quick search and obtain the optimal solution or the approximate optimal solution. Having no determined trajectory, the particle with quantum behaviors can appear in any position randomly, which is consistent with the randomness of human thinking; therefore, the search method of QPSO is similar to the human intelligence. Compared with standard PSO, QPSO has stronger global optimization ability and fewer algorithm parameters, making it easier to realize and control [2].

This paper has made a comprehensive analysis of the theory and research status of image denoising and it has

also made an in-depth research of the idea and realization of image denoising based on wavelet analysis and intelligent optimization. Moreover, it discusses and compares the selection of the wavelet decomposition level, the wavelet basis, the wavelet coefficient estimation method and threshold, integrates the wavelet transform denoising and QPSO and optimizes the parameters used in wavelet transform and it has achieved better image denoising effects.

2 Wavelet image denoising

2.1 THEORETICAL BASIS OF WAVELET ANALYSIS

2.2.1 Continuous wavelet transforms (CWT)

Assuming that $\psi(t)$ is a square integrable function, namely $\psi(t) \in L^2(R)$, and $\hat{\psi}(0) = 0$ when its Fourier transform $\omega = 0$, namely $\int_{-\infty}^{+\infty} \psi(t) dt = 0$, $\psi(t)$ is called a basic wavelet or a mother wavelet. Scale and translate the generating function and get:

$$\psi_{a,b}(t) = \frac{1}{\sqrt{|a|}} \psi\left(\frac{t-b}{a}\right), a, b \in R, a \neq 0 \quad (1)$$

$\psi_{a,b}(t)$ is called wavelet function, wavelet for short. In this formula, a is scale factor and b is translation factor. The variable a reflects the scale (width) of the function while the variable b detects translation position of the wavelet function in Axel t . In general, the energy of the mother

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wavelet $\psi(t)$ focuses on the origin while the energy of the wavelet function $\psi_{a,b}(t)$ on point b . The typical wavelet function is indicated in Figure 1 [3].

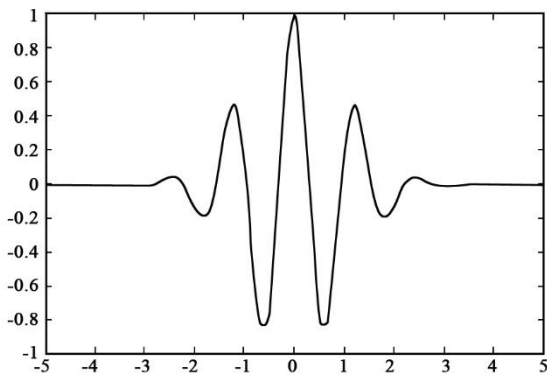


FIGURE 1 Wavelet function

In practical applications, it is generally assumed that $a > 0$. At this time, in the definition of the wavelet function $\psi_{a,b}(t)$, the scale factor a is supposed to stretch the basic wavelet $\psi(t)$. The bigger a is, the broader $\psi(t/a)$ is. The wavelet duration broadens with the increase of a . The range of the wavelet function is inversely proportional to the reduce of \sqrt{a} ; however, the wavelet shape remains unchanged. Stretch the basic wavelet $\psi(t)$ to get $\psi(t/a)$ and form a group of basic functions. In the large a , search large characteristics with the expanded basic function while as for the small a , search the detail characteristics. The role of $1/\sqrt{a}$ in $\psi_{a,b}(t)$ is to keep the energy of the wavelet $\psi_{a,b}(t)$ with different values the same, namely $\|\psi(t)\|_2 = \|\psi_{a,b}(t)\|_2$.

The CWT of any function $f(t) \in L^2(R)$ is:

$$W_f(a,b) = \langle f, \psi_{a,b} \rangle = \frac{1}{\sqrt{|a|}} \int_R f(t) \psi^*\left(\frac{t-b}{a}\right) dt \quad (2)$$

In this Equation, $\psi^*(t)$ means the complex conjugate of (t) .

If this wavelet is an orthogonal wavelet, its reconstruction Equation (inverse transformation) is:

$$f(t) = \frac{1}{C_\psi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{1}{a^2} W_f(a,b) \psi\left(\frac{t-b}{a}\right) ab db \quad (3)$$

The energy must be kept proportional in wavelet transform, namely:

$$C_\psi \int_R |f(t)|^2 dt = \int_R \frac{da}{a^2} \int_R |W_f(a,b)|^2 ab \quad (4)$$

Since the wavelet produced by the basic wavelet $\psi(t)$ in the wavelet transform plays as an observation window of the analyzed signal, $\psi(t)$ shall also meet the constraints of the general function

$$\int_{-\infty}^{+\infty} |\psi(t)| dt < \infty \quad (5)$$

So, $\hat{\psi}(\omega)$ is a continuous function, and it means that in order to meet the reconstruction conditions, $\hat{\psi}(\omega)$ must be zero at the origin, namely:

$$\hat{\psi}(0) = \int_{-\infty}^{+\infty} \psi(t) dt = 0 \quad (6)$$

In order to make the realization of signal reconstruction stable, the Fourier transform of wavelet $\psi(t)$ shall also satisfy the following stability conditions apart from the perfect reconstruction condition [4]:

$$A \leq \sum_{-\infty}^{+\infty} |\hat{\psi}(2^{-j}\omega)|^2 \leq B \quad (7)$$

In this Equation, $0 < A \leq B < \infty$.

2.1.2 Discrete wavelet transform (DWT)

In practical applications, discrete wavelet transform (DWT) is mostly considered. Discretization refers to the discretization of scale and displacement, aimed for the continuous scale parameter a and the continuous translation parameter b instead of time t .

Generally speaking, take the values $a=2^{-j}$ and $b=k2^{-j}$ with $k, j \in Z$ for the discretization formulas of the scale parameter a and the translation parameter b in CWT, then the discrete wavelet function $\psi_{j,k}(t)$ is:

$$\psi_{j,k}(t) = 2^{j/2} \psi\left(\frac{t-k2^{-j}}{2^{-j}}\right) = 2^{j/2} \psi(2^{-j}t-k) \quad (8)$$

The coefficients of DWT can be expressed as:

$$C_{j,k} = \langle f, \psi_{j,k} \rangle = \int_{-\infty}^{+\infty} f(t) \psi_{j,k}^*(t) dt \quad (9)$$

Its reconstruction Equation is:

$$f(t) = c \sum_{-\infty}^{+\infty} \sum_{-\infty}^{+\infty} C_{j,k} \psi_{j,k}(t) \quad (10)$$

In this Equation, C is an irrelevant constant to signal and appropriate j and k need to be selected to guarantee the accuracy of the reconstruction signal. Obviously, there need to be as many network points as possible because if there are few network points, there are few wavelet function $\psi_{j,k}(t)$ and discrete wavelet coefficient $C_{j,k}$ to be used and the signal reconstruction is not very precise [5,6].

2.2 BASIC METHODS OF WAVELET THRESHOLD DENOISING

2.2.1 Principle of threshold denoising

The basic idea of wavelet threshold denoising proposed by Donoho is that when $w_{j,k}$ is smaller than a certain critical threshold, the wavelet coefficient at this time is mainly caused by noise and it shall be abandoned and that when

$w_{j,k}$ is bigger than this critical threshold, the wavelet coefficient at this moment is mainly caused by signal and $w_{j,k}$ shall be directly saved the at this time (hard thresholding method) or compress a certain fixed amount towards zero (soft thresholding method). Then get the denoised signal by using new wavelet coefficient to conduct wavelet reconstruction. This method can be realized by the following three steps:

1) Make wavelet transform to the noisy signal $f(t)$ and get a group of wavelet decomposition coefficient $w_{j,k}$.

2) Perform threshold processing to the decomposed wavelet coefficient $w_{j,k}$; get the estimated wavelet coefficient $\bar{w}_{j,k}$ and make $w_{j,k} - u_{j,k}$ as small as possible ($u_{j,k}$ is the wavelet transform coefficient without noisy signal).

3) Reconstruct the estimated wavelet coefficient $\bar{w}_{j,k}$ and get the estimated signal $\bar{f}(t)$, namely the denoised signal.

It needs to be pointed out that in the wavelet threshold denoising, it is most important to select the threshold function and thresholds.

2.2.2 The selection of threshold function

The threshold function is related to the continuity and precision of the reconstruction signal and it also has a huge impact on the wavelet denoising effects. Currently, the selection of the threshold is mainly divided into hard thresholding processing and soft thresholding processing. Soft thresholding processing is to compare the absolute value of the signal and its threshold. When the absolute value is smaller than or equal to the threshold, make it zero; otherwise, compress towards zero and make it the difference between the point and the threshold. Hard thresholding processing compares the absolute value of the signal and its threshold, and make it zero if it is smaller than or equal to the threshold and keep it unchanged if it is bigger than the threshold. The in-continuity of hard thresholding processing makes the denoised signal still have obvious noise. Although soft thresholding method has good continuity, there are constant deviations between the estimated wavelet coefficient and the wavelet coefficient with noisy signal and when the noisy signal is irregular, it will be too smooth. The expressions of these two threshold functions are as follows.

The hard thresholding function is:

$$\bar{w}_{j,k} = \begin{cases} w_{j,k}, & |w_{j,k}| > \lambda \\ 0, & |w_{j,k}| \leq \lambda \end{cases} \quad (11)$$

The soft thresholding function is:

$$\bar{w}_{j,k} = \begin{cases} \text{sgn}(w_{j,k})(|w_{j,k}| - \lambda), & |w_{j,k}| > \lambda \\ 0, & |w_{j,k}| \leq \lambda \end{cases} \quad (12)$$

In this Equation, $\text{sgn}(*)$ is the sign function, namely

$$\text{sgn}(n) = \begin{cases} 1, n > 0 \\ -1, n < 0 \end{cases} \quad (13)$$

Therefore, hard thresholding method can better preserve the local characteristics such as image edge, but the visual distortion like ringing and pseudo-Gibbs phenomenon while although soft thresholding processing is relatively smooth, it may cause edge blur and other distortion phenomena [7].

2.2.3 The shortcomings of soft & hard threshold functions

Though soft and hard thresholding methods have been extensively used and have obtained better effects, they have many weaknesses, too:

1) Soft Thresholding Method: Although soft threshold function is continuous in wavelet domain without any discontinuous points, its derivative is not continuous; therefore, it is difficult to seek higher derivative. Besides, the wavelet coefficient with soft threshold bigger than the threshold adopts constant compression, which is inconsistent with the trend that noise component gradually decreases with the increase of wavelet coefficient.

2) Hard Thresholding Method: Hard threshold function is not continuous in the entire wavelet domain and there are discontinuous points between λ and $-\lambda$, which has some conflicts with the practical applications where derivation is frequently sought in threshold function and which has certain limitations. In the meanwhile, it only processes the wavelet functions smaller than threshold instead of those with bigger threshold, which is also inconsistent with the fact that there is noisy signal interruption in the wavelet coefficients bigger than the threshold.

In the wavelet threshold denoising, it is very critical to select the threshold. If the threshold is small, the denoised image signal is relatively closed to the input; however, there are many residual noises. If the threshold is big, the wavelet coefficient with many zeros can be obtained. The reconstructed image under soft threshold policy becomes blurred while that under hard threshold policy includes more false edges. In the wavelet threshold denoising, the selection of threshold directly affects the filtering effects [8].

3 Quantum-behaved particle swarm optimization (QPSO)

3.1 QUANTUM-BEHAVED PARTICLE SWARM OPTIMIZATION

QPSO describes the status of the particle through wave function and the probability density function for the

particle to appear in a certain point of the space can be seeked through Schrodinger Equation. Finally, use Monte Carlo Random Simulation to obtain the position equation of the particle and this equation can randomly determine the position of the moving particle in one-dimensional DELTA Momentum centered by point p .

$$x^k = p \pm \frac{L}{2} \ln\left(\frac{1}{u}\right). \quad (14)$$

In this Equation, u is a random number evenly distributed within $[0,1]$ and Z can be expressed as follows:

$$L^{k+1} = 2\beta |mbest - x^k|. \quad (15)$$

Finally, the iteration equation of QPSO can be expressed as follows:

$$mbest^k = \frac{1}{M} \sum_{i=1}^M p_i^k = \left[\frac{1}{M} \sum_{i=1}^M p_{i1}^k, \frac{1}{M} \sum_{i=1}^M p_{i2}^k, \dots, \frac{1}{M} \sum_{i=1}^M p_{iD}^k \right] \quad (16)$$

$$p_{id}^k = \varphi \cdot p_{id}^k + (1-\varphi) \cdot p_{gd}^k. \quad (17)$$

$$x_{id}^{k+1} = p_{id}^k \pm \beta \cdot |mbest^k - x_{id}^k| \cdot \ln\left(\frac{1}{u}\right). \quad (18)$$

In the Equation, M is the population size of the particle; D is the dimensions of the particle; $mbest$ is the mean optimal positions of all particles and φ is a random number evenly distributed within $[0,1]$. β is the only coefficient in QPSO. When $\beta > 0.5$, Equation (18) is the sum of two items and when $\beta \leq 0.5$, Equation (18) is reduction of these two items. Generally, β can be obtained from Equation (19):

$$\beta = m - (m - n) \times \frac{k}{Maxtimes} \quad (19)$$

In this Equation, $m=1$; $n=0.5$ and $Maxtimes$ is the maximum iterations.

The realization steps of QPSO are as follows [9,10]:

- 1) Initialize every particle and the parameters.
- 2) Calculate the corresponding fitness value to every particle, namely $F(x_i^k)$.
- 3) Compare the fitness value of the current particle and the individual optimal value (assuming that the optimal value is the minimum value). If $F(x_i^k) < f(p_i^{k-1})$, update the individual optimal position, namely $p_i^k = x_i^k$; otherwise, keep it unchanged.
- 4) Calculate the mean optimal position of the particle swarm according to Equation (16).
- 5) Check whether to update the global optimal position p_g^{k-1} . If $f(p_i^k) < f(p_g^{k-1})$, update the global optimal position; otherwise, keep it unchanged.
- 6) Update the particle positions according to Equations (17) and (18).

- 7) Judge whether to meet the iteration termination conditions. If it does, end the calculation and output the results; otherwise, return to Step 2.

The algorithm flow chart of QPSO is indicated as Figure 2:

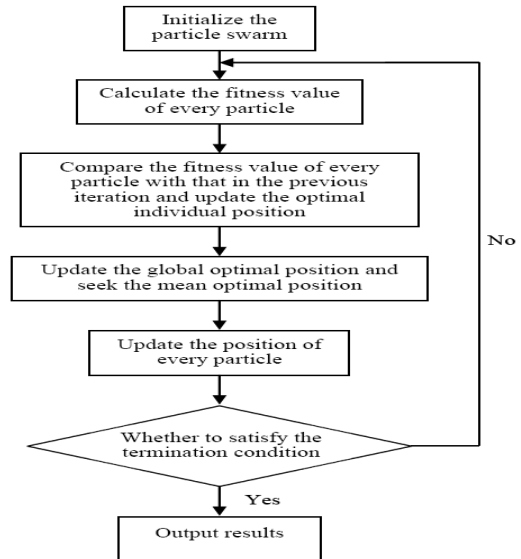


FIGURE 2 The flow chart of quantum-behaved particle swarm optimization

4 Image denoising based on wavelet analysis and quantum-behaved particle swarm optimization

4.1 THE DETERMINATION OF FITNESS FUNCTION

Perform two-dimensional wavelet coefficient transform; define the initial position of the particle as any point randomly distributed in every high-frequency sub-band with every sub-band as a solution space; give the particle a random initial velocity and search the optimal solution in the given sub-band by using PSO to make the fitness function $fitness(\lambda)$ smallest with λ to be the optimal threshold.

In the k -th-level decomposition sub-band, there is an optimal threshold $\lambda_{k,opt}$ when the mean square error (MSE) between the wavelet coefficient without interference noise and the denoised wavelet coefficient and the selection of threshold λ can be defined through a function:

$$R_k(\lambda) = \frac{1}{N_k} \|W_{k,\lambda} - V_k\|^2 \quad k = 1, 2, \dots, n \quad (20)$$

In this Equation, N_k is the number of wavelet coefficient in the k th-level decomposition sub-band; $W_{k,\lambda}$ is the thresholding wavelet coefficient matrix and V_k is the wavelet coefficient matrix without noise influence. This paper uses Equation (20) as the fitness function for the algorithm of this paper.

4.2 THE DENOISING ALGORITHM PROCEDURE OF WAVELET INTELLIGENT OPTIMIZATION

The specific steps of the algorithm of this paper are classified as follows:

1) Determine the decomposition levels according to the variance estimate of the noise.

2) Assume that the particle swarm size is $M=60$; the maximum iterations are $N=500$; the particle dimensions are 3 and the independent threshold of different levels is λ . Set the learning factors c_1 & c_2 are 0.9-1; the inertia weight w is 0.9-1; the particle search space is $[0,800]$ and the particle velocity $v \in [1,8]$. Initialize the velocity of the particle swarm and the particles and read in image.

3) Calculate the fitness of various particles according to Equation (20).

4) Compare the fitness and determine the individual optimal value point and the global optimal value point. If the fitness value $< f_{pbest}$, then f_{pbest} = the fitness value and the individual optimal value position $P_{best} = x_i$; otherwise keep P_{best} unchanged and if $f_{pbest} < f_{gbest}$, then $f_{gbest} = f_{pbest}$, $g_{best} = p_{pbest}$, otherwise keep g_{best} unchanged. Here, P_{best} is the individual optimal value of the particle; g_{best} is the global optimal value; f_{pbest} is the fitness to obtain the individual optimal value and f_{pbest} is the fitness when obtaining the global optimal value.

5) Update the velocity and position of the particles according to Equations (16), (17) and (18) and judge whether the updated velocity is within the limited range.

6) If the minimum fitness value is smaller than 0.001 after implementing the algorithm to the maximum iteration or three continuous iterations, the algorithm ends.

7) Substitute the threshold from the training to implement image denoising.

8) End the algorithm.

5 Simulation experiment and result analysis

5.1 EVALUATION METHODS FOR IMAGE QUALITY

Objective evaluation methods are to use the errors to restore the image deviates from the original image to balance the quality of image restoration and the most

commonly-used methods include: mean square error (*MSE*), signal noise ratio (*SNR*), peak mean square error (*PMSE*) and peak signal noise ratio (*PSNR*). The objective evaluation methods can only generally reflect the grayscale differences between the original image and the restored image.

MSE is the *MSE* between the original signal and the denoised estimated signal and its definition is:

$$MSE = \frac{1}{M \times N} \sum_{m=1}^M \sum_{n=1}^N (\hat{X}(m,n) - X(m,n))^2 \quad (21)$$

In this Equation, $X(m,n)$ is the original signal; $\hat{X}(m,n)$ is the estimated signal after wavelet denoising; M and N are the rank dimensions of the image respectively. The smaller the denoised *MSE* means that the approximation between the denoised image and the original image is higher and that the denoising effects are better. The *MSE* of the peak is generally expressed equivalent *SNR*, namely *PSNR*:

$$PSNR = -10 \lg \left(\frac{MSE}{255^2} \right). \quad (22)$$

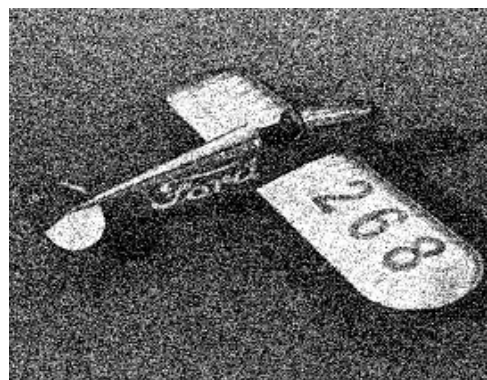
Image quality evaluation methods have different measurement standards. Since the accuracy of human visual system characteristics can't be described through quantitative approaches, quantitative description can't be made to the subjective evaluation methods and it is greatly affected by human factors, but it can reflect human visual characteristics. *PSNR* can make quantitative description to the image denoising quality. Therefore, this paper uses *SNR* of the image to analyze the effects of various denoised images.

5.2 EXPERIMENTAL ANALYSIS

Taking the standard liftingbody (512×512) image as example, the algorithm of this paper adds the impulse noise with a noise intensity of 0.2 to observe the effects of various denoising algorithms. And their specific effects can be seen in Figure 3.



a) Original image



b) Noise image

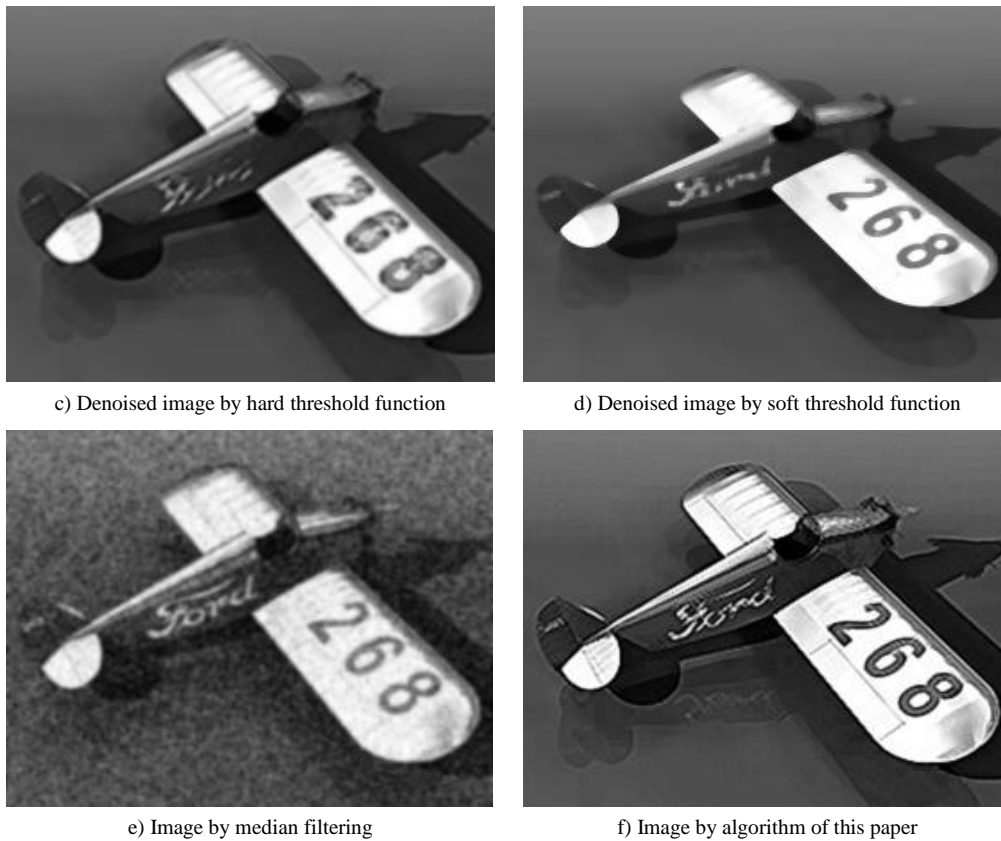


FIGURE 3 Comparison of image denoising effects

Table 1 is to calculate its peak signal to noise ration by adding different noise coefficients.

TABLE 1 Comparison table of peak signal to noise ratio (PSNR) of image in different filtering algorithms (Unit db)

Noise intensity	Noisy image	Median filtering	Hard thresholding function denoising	Soft thresholding function denoising	Algorithm of this paper
0.05	18.9763	21.3513	23.4345	27.5848	32.3492
0.1	15.9457	20.3748	22.5671	26.3729	31.3482
0.2	13.8398	20.0118	22.495	25.2493	30.3849
0.3	12.0337	19.3712	21.3748	24.4852	29.3843
0.4	11.1637	18.5548	20.4749	23.4194	28.3832
0.5	9.9373	17.4592	19.3472	21.4859	26.3757

By observing the effect figures of the above two images in different filtering algorithms, we can see that different algorithms have different effects and it can be seen from the comparison that the algorithm of this paper is obviously better than other algorithms. We can analyze the performance of peak signal to noise ratio (PSNR) of the image by different algorithms from the data in Table 1 and it can be seen that with the continuous improvements of noise intensity, the PSNR of the image reduces continuously. It can be seen from the statistics in the table that the effect of median filtering algorithm is bad and the effects of hard/soft threshold functions are better than that of median filtering; however, their effects are gradually closed to that of median filtering and it is not very ideal in

big noise. Therefore, the denoising effect by the algorithm of this paper is obviously better than other algorithms and it has certain practical value.

6 Conclusions

This paper investigates wavelet analysis and image denoising principle and realizes the image denoising based on wavelet analysis and quantum-behaved particle swarm optimization. The experimental results shows that while removing the image noise, this method effectively maintains the detail information and texture features of the image, presents excellent denoising ability and has certain practical value.

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